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# Some observations on dynamic vibrations of homogeneous incompressible elastic shells,

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By Cinelli's method (1965, 1966), a suitable alternative form of the method of Hankel's transform Chatterjee (1968), claimed to find the transient displacements and stressed for 'dynamic' (?) vibrations of homogeneous incompressible elastic spherical and cylindrical shells as a series of Cinelli's functions which are a linear combination of two Bessel functions of the same order. Due to incompressibility, *i.e.* the vanishing of volume dilatation, the only non-zero displacement, $u(r, t)$, satisfies a partial differential equation of the first order involving $\frac{\partial u}{\partial r}$ and so $\frac{\partial^2 u}{\partial r^2}$ and $\frac{\partial u}{\partial r}$ in the equation of motion are replaceable by expressions involving $u(r,t)$ only. In some recent general discussions of solutions of differential equations by methods of integral transforms (Dutta & Debnath 1965, 1967), it was explained clearly that a differential equation involving some differential operator $L$ can be solved by an integral transform associated with an irreducible differential operator $L$ (if $L = f(L)$) where, $f(z)$ is a polynomial (or an integral function) of $z$ with constant coefficients. Then, a differential equation is solvable by Cinelli's method if its differential operator is at least of the second order. But in the case of incompressible elastic shells, the equation of motion is reducible to one of order less than two regarding the partial derivative with respect to $r$ and is not solvable by Cinelli's method. The expression for displacement in terms of Cinelli's functions (Chatterjee, 1968) does not satisfy the differential relation signifying the incompressibility. This point has been clarified by some easy discussions from the theory of operators. Straight calulation implies the impossibility of time-dependent radial displacements for the problems discussed.

In the case of incompressible spherical shells for radial displacements in polar coordinates

$$u_r = u(r,t),\ u_\theta = 0,\ u_\phi = 0 \qquad \dots (1)$$

due to incompressibility, one gets

$$\frac{\partial u}{\partial r} + \frac{2u}{r} = 0 \qquad \dots (2)$$

After proper choice of elastic constants the equation of motion is (Love 1944)

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} - \frac{2u}{r^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \quad (c \text{ being a constitutive constant}) \quad \ldots(3)$$

with initial condition,

$$\frac{\partial u}{\partial t} = 0 \quad \text{at} \quad t = 0. \quad \ldots(4)$$

Due to the equation (2), the equation (3) reduces to the equation,

$$0 = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}, \quad \text{or} \quad u(r,t) = A(r)t + B(r) \quad \ldots(5)$$

By (3), $A(r) \equiv 0$ and $u(r,t) = B(r)$ ...(6)

Even if the initial condition is not used, the fact that the linear theory of elasticity is a theory of small displacements leads to (5).
Then from (1) and (4)

$$\frac{B'(r)}{B(r)} = -\frac{2}{r} \quad \text{or} \quad u(r,t) = B(r) = Cr^{-2} \quad (C \text{ being a constant}) \quad \ldots(7)$$

Thus, the loads in the outer and the inner surfaces must be in a ratio $a^3 : b^3$, $a$ and $b$ being the inner and the outer radii. The equation (7) shows that there cannot be a radial vibration and the loads on the boundary surfaces cannot be arbitrary as asserted by Chatterjee (1968).

In case of incompressible cylindrical shells for displacements in cylindrical coordinates,

$$u_r = u(r,t), \; u_\theta = 0, \; u_z = 0 \quad (1)$$

the equation of motion after proper choice of elastic constants (Love, 1944) is

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \quad \ldots(3)$$

with the initial condition,

$$\frac{\partial u}{\partial t} = 0, \quad \text{at} \quad t = 0 \quad (4)$$

Proceeding as in the previous case, one gets

$$u(r,t) = C'r^{-1} \quad (C' \text{ being a constant}) \quad \ldots(7)$$

The loads on the boundary surfaces are in the ratio $a^2 : b^2$, $a$ and $b$ being the outer and the inner radii. The concluding comments are the same as in the previous case.

To get a clear idea about the nature of errors, let us denote

$$l_{1r} \equiv \frac{\partial}{\partial r} + \frac{2}{r}, \; l'_{1r} \equiv \frac{\partial}{\partial r} + \frac{1}{r}, \qquad \ldots(8)$$

$$l_{2r} \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{4}{r^2}, \quad l'_{2r} \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}. \qquad \ldots(9)$$

The linear operators are defined on a set of functions having continuous derivative of the second order in $[a, b]$ which, after topologising in the usual way, is denoted by $D_2[a,b]$ (Shilov, 1965). It is easy to see that

$$l_{2r} \equiv \left( l_{1r} - \frac{3}{r} \right), l'_{1r} \equiv \left( l'_{1r} - \frac{1}{r} \right) l'_{1r} \qquad \ldots(10)$$

Relations (10) show that the solutions of $l_{1r}\, u = 0$ and $l_{1r}\, u = 0$ are solutions of $l_{2r}\, u = 0$ and $l'_{2r}\, u = 0$, respectively. But the converse is not true. Nonnull images of the set points in $D_2[a, b]$ for which $\left(l_{1r} - \frac{3}{r}\right)u = 0$ and $\left( l'_{1r} - \frac{1}{r}\right) u = 0$, respectively, with respect to $(l_{1r})^{-1}$ and $(l'_{1r})^{-1}$ are solutions of $l_{2r}u = 0$, and $l_2'{}_r u = 0$, respectively, but evidently $l_{1r}\, u \neq 0$ and $l'_{1r} u \neq 0$.

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# Decay characteristics of CaS (Zr, Di) phosphors

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Phosphorescence decay characteristics of CaS phosphors activated with Zr and Di (Pr+Nd) impurities have been studied to gather information about the type of phosphorescence decay, value of time constant of decay and the effective trap depths contributing to the phosphorescence in this system of phosphors.